حل التمرين 1

1) نحسب مايلي:

$a = (3 - \sqrt{5})^2 = 9 - 6\sqrt{5} + 5$

أي:

$\boxed{a = 14 - 6\sqrt{5}}$

$b = (2\sqrt{3} + 3\sqrt{2})^2 = 12 + 12\sqrt{6} + 18$

أي:

$\boxed{b = 30 + 12\sqrt{6}}$

أي:

$C = (\sqrt{2} + 4)^2 = 2 + 8\sqrt{2} + 16$

أي:

$C = 18 + 8\sqrt{2}$

$d = \left(\frac{\sqrt{2}}{\sqrt{3} + \sqrt{2}}\right)^{-2} = \left(\frac{\sqrt{3} + \sqrt{2}}{\sqrt{2}}\right)^2$

$d = \frac{3 + 2\sqrt{6} + 2}{2}$

أي:

$d = \frac{5 + 2\sqrt{6}}{2}$

بطريقة أخرى:

$d = \left(\frac{\sqrt{2}}{\sqrt{3} + \sqrt{2}}\right)^{-2} = \left(\frac{\sqrt{2}(\sqrt{3} - \sqrt{2})}{(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})}\right)^{-2}$

$A = 8\sqrt{5}$:أي $A = 12\sqrt{5} - 6\sqrt{5} + 2\sqrt{5}$

$B = \sqrt{32} + 3\sqrt{2} - \sqrt{72}$

$B = \sqrt{16 \times 2} + 3\sqrt{2} - \sqrt{36 \times 2}$

$B = \sqrt{2}$ $B = 4\sqrt{2} + 3\sqrt{2} - 6\sqrt{2}$

$C = 2\sqrt{\frac{3}{4}} + \sqrt{27} + \frac{1}{2}\sqrt{12} - 3\sqrt{\frac{75}{9}}$

$C = \frac{2\sqrt{3}}{2} + \sqrt{9 \times 3} + \frac{1}{2}\sqrt{4 \times 3} - \frac{3\sqrt{25 \times 3}}{3}$

$C = \sqrt{3} + 3\sqrt{3} + \frac{1}{2}.2\sqrt{3} - 5\sqrt{3}$

$C = 0$:أي $C = \sqrt{3} + 3\sqrt{3} + \sqrt{3} - 5\sqrt{3}$

$D = \sqrt{0,98} + \sqrt{1,62} - 4\sqrt{0,72}$

$D = \sqrt{2 \times 0,49} + \sqrt{2 \times 0,81} - 4\sqrt{2 \times 0,36}$

$D = \sqrt{2}\sqrt{49 \times 10^{-2}} + \sqrt{2}\sqrt{81 \times 10^{-2}} - 4\sqrt{2}\sqrt{36 \times 10^{-2}}$

$D = 7 \times 10^{-1}\sqrt{2} + 9 \times 10^{-1}\sqrt{2} - 24 \times 10^{-1}\sqrt{2}$

$D = 10^{-1}\sqrt{2}(+7 + 9 - 24)$

$D = \frac{-8}{10}\sqrt{2}$:أي $D = -8 \times 10^{-1}\sqrt{2}$

أي: $D = \frac{-4\sqrt{2}}{5}$

$E = \sqrt{4a} + \sqrt{9a} + 2\sqrt{a} - \frac{7}{5}\sqrt{25a}$

$E = 2\sqrt{a} + 3\sqrt{a} + 2\sqrt{a} + -\frac{7}{5}.5\sqrt{a}$

$E = (2 + 3 + 2 - 7)\sqrt{a}$

$E = 0$:أي $E = (7 - 7)\sqrt{a}$

$F = \sqrt{5^2 + 3^2 + 4^2 - 1}$

تعني: $F = \sqrt{25 + 9 + 16 - 1}$

$F = 7$:أي $F = \sqrt{49}$

$H = \sqrt{3 \times 5^2 \times 7 \times 21}$

$H = \sqrt{3 \times 5^2 \times 7 \times 7 \times 3} = \sqrt{3^2 \times 5^2 \times 7^2}$

$H = 3 \times 5 \times 7 = 105$:أي $H = \sqrt{(3 \times 5 \times 7)^2}$:أي

$I = \frac{\sqrt{0,0032}}{\sqrt{0,18}} = \sqrt{\frac{32 \times 10^{-4}}{18 \times 10^{-2}}}$

$I = \sqrt{\frac{16 \times 10^{-4} \times 10^{2}}{9}} = \sqrt{\frac{4^2 \times 10^{-2}}{3^2}} = \sqrt{\left(\frac{4 \times 10^{-2}}{3}\right)^2}$

$d = \left(\frac{\sqrt{6} - 2}{3 - 2}\right)^{-2} = \left(\frac{\sqrt{6} - 2}{1}\right)^{-2}$

$d = \left(\frac{1}{\sqrt{6} - 2}\right)^2 = \frac{1}{6 - 4\sqrt{6} + 4} = \frac{1}{10 - 4\sqrt{6}} = \frac{1}{2(5 - 2\sqrt{6})}$

تعني: $d = \frac{5 + 2\sqrt{6}}{2(5 - 2\sqrt{6})(5 + 2\sqrt{6})}$

تعني: $d = \frac{5 + 2\sqrt{6}}{2(25 - 24)}$:أي $d = \frac{5 + 2\sqrt{6}}{2}$

$e = (1 + \sqrt{2} - \sqrt{7})(1 - \sqrt{2} + \sqrt{7})$

$e = 1 - \sqrt{2} + \sqrt{7} + \sqrt{2} - 2 + \sqrt{14} - \sqrt{7} + \sqrt{14} - 7$

أي: $e = 2\sqrt{14} - 8$

بطريقة أخرى:

$e = \left[1 + (\sqrt{2} - \sqrt{7})\right]\left[1 - (\sqrt{2} - \sqrt{7})\right]$

$e = 1^2 - (\sqrt{2} - \sqrt{7})^2 = 1 - (2 - 2\sqrt{14} + 7)$

$e = 1 - 9 + 2\sqrt{14}$

$e = 2\sqrt{14} - 8$

أي: $f = \sqrt{2}(\sqrt{3} - \sqrt{2}) + \sqrt{3}(\sqrt{3} - \sqrt{2}) - 1$

إذن: $f = \sqrt{6} - 2 + 3 - \sqrt{6} - 1$

$f = \sqrt{6} - \sqrt{6} + 3 - 3 = 0$

تعني: $i = (\sqrt{5} - 1)^2 - 2(\sqrt{5} + 1)^2 + 6\sqrt{5}$

$i = (5 - 2\sqrt{5} + 1) - 2(\sqrt{5^2} + 2\sqrt{5} + 1) + 6\sqrt{5}$

$i = 6 - 2\sqrt{5} - 2(6 + 2\sqrt{5}) + 6\sqrt{5}$

$i = -6$ $i = -6 - 6\sqrt{5} + 6\sqrt{5}$

2) نبسط ما يلي:

تذكير:

إذا كان a و b عددين حقيقين موجبين فإن :

$\sqrt{ab} = \sqrt{a}.\sqrt{b}$ $\sqrt{a^2} = a$

$A = 3\sqrt{80} - 2\sqrt{45} + \sqrt{20}$

$A = 3\sqrt{16 \times 5} - 2\sqrt{9 \times 5} + \sqrt{4 \times 5}$

$A = 3(4\sqrt{5}) - 2(3\sqrt{5}) + 2\sqrt{5}$

$$I = \frac{4 \times 10^{-1}}{3} = \frac{4}{30}$$ أي: $$\boxed{I = \frac{2}{15}}$$ أي:

$$J = \frac{\sqrt{49 \times 10^8}}{\sqrt{10000}} = \sqrt{\frac{7^2 \times 10^8}{10^4}}$$

$$J = \sqrt{7^2 \times 10^4} = \sqrt{(7 \times 10^2)^2}$$ أي: $$J = 7 \times 10^2$$

يعني: $$\boxed{J = 700}$$

3) حساب: $$x^2 + \frac{1}{x^2}$$

$$x + \frac{1}{x} = \sqrt{3}$$ تعني: $$\left(x + \frac{1}{x}\right)^2 = \sqrt{3^2}$$

$$x^2 + 2\cancel{x}.\frac{1}{\cancel{x}} + \frac{1}{x^2} = 3$$

تعني: $$x^2 + \frac{1}{x^2} + 2 = 3$$ تعني: $$x^2 + \frac{1}{x^2} = 3 - 2$$

أي: $$\boxed{x^2 + \frac{1}{x^2} = 1}$$