حساب التكامل A

لدينا
$$A=\int_{-1}^{1}(x+1)^2\,dx$$
$$=\left[\frac{1}{3}(x+1)^3\right]_{-1}^{1}$$
$$=\frac{2^3}{3}$$

إذن $A=\frac{8}{3}$

حساب التكامل B

لدينا
$$B=\int_{1}^{4}\frac{1+x}{\sqrt{x}}\,dx$$
$$=\int_{1}^{4}\left(\frac{1}{\sqrt{x}}+\frac{x}{\sqrt{x}}\right)dx$$
$$=\int_{1}^{4}\frac{1}{\sqrt{x}}\,dx+\int_{1}^{4}\sqrt{x}\,dx$$

$$= [2\sqrt{x}]_1^4 + \left[\frac{2}{3}(\sqrt{x})^3\right]_1^4$$

$$= 2 + \frac{2}{3}(2^3 - 1)$$

إذن $B = \frac{20}{3}$

حساب التكامل C

لدينا

$$C = \int_0^{\frac{\pi}{2}} \cos\left(2x - \frac{\pi}{3}\right) dx$$

$$= \left[\frac{1}{2}\sin\left(2x - \frac{\pi}{3}\right)\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\sin\left(\frac{2\pi}{3}\right) - \frac{1}{2}\sin\left(-\frac{\pi}{3}\right)$$

إذن $C = \frac{\sqrt{3}}{2}$

حساب التكامل D

لدينا $x \longmapsto x^5$ هي الدالة المشتقة للدالة u المعرفة بما يلي :

$$u(x) = \frac{1}{6}x^6$$

ومنه

$$D = \int_1^e u'(x) \ln x \, dx$$

$$= [u(x)\ln x]_1^e - \int_1^e u(x) \ln'(x) \, dx$$

$$= \left[\frac{1}{6}x^6 \ln x\right]_1^e - \int_1^e \frac{1}{6}x^6 . \frac{1}{x} dx$$

$$= \frac{1}{6}e^6 - \frac{1}{6}\int_1^e x^5 dx$$

$$= \frac{1}{6}e^6 - \frac{1}{6}\left[\frac{1}{6}x^6\right]_1^e$$

$$= \frac{1}{6}e^6 - \frac{1}{36}e^6 + \frac{1}{36}$$

إذن $D = \frac{1}{6}e^6 - \frac{1}{36}e^6 + \frac{1}{36} \quad = \frac{5e^6 + 1}{36}$

حساب التكامل E

نضع $x = \sin t$ $\left(0 \leq t \leq \frac{\pi}{2}\right)$ ويكون لدينا $dx = \text{cost.dt}$

وإذا كان $x = 0$ فإن $t = 0$

وإذا كان $x = \frac{1}{2}$ فإن $t = \frac{\pi}{6}$

ومنه

$$E = \int_0^{\frac{\pi}{6}} \frac{\sin^2 t}{\sqrt{1-\sin^2 t}} \cos t \, . \, dt$$

$$= \int_0^{\frac{\pi}{6}} \frac{\sin^2 t}{\cos t} \cos t \, . \, dt$$

$$= \int_0^{\frac{\pi}{6}} \sin^2 t \, dt$$

$$= \int_0^{\frac{\pi}{6}} \frac{1-\cos 2t}{2} dt$$

$$= \frac{1}{2}\left[t - \frac{1}{2}\sin 2t\right]_0^{\frac{\pi}{6}}$$

$$= \frac{1}{2}\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$$

إذن

$$E = \frac{\pi}{12} - \frac{\sqrt{3}}{8}$$